On the packet ordering problem

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Problem Definition

- Packet transmission between two nodes, $\mathcal{A}$ and $\mathcal{B}$
- Suppose $\mathcal{A}$ sends N packets to $\mathcal{B}$
- Assume all N packets arrive at $\mathcal{B}$ - no loss
- But they arrive out of order
- Is there a metric that quantifies this 'out-of-orderedness'?

Send and receive order

- Packet sequence from $\mathcal{A}$: $\mathcal{X}_{\mathcal{A}} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

- Packet set received at $\mathcal{B}$: $\mathcal{X}_{\mathcal{B}} = \{3, 2, 4, 6, 5, 9, 7, 1, 10, 8\}$

- Which members of $\mathcal{X}_{\mathcal{B}}$ are in order and which are not?

- Consider *ascending subsequences* of $\mathcal{X}_{\mathcal{B}}$ of length $m \leq 10$

$$m = 3: \quad \mathcal{S} \equiv \{2, 4, 6\}$$
$$\mathcal{S} \equiv \{4, 7, 10\}$$
$$\mathcal{S} \equiv \{5, 9, 10\}$$

Longest ascending subsequences

- For given $\mathcal{X}_{\mathcal{B}}$, there exists some $m = m_{max}$ such that no *ascending subsequence* is of length greater than m_{max}

- Longest *ascending subsequences* of

$$\mathcal{X}_{\mathcal{B}} = \{3, 2, 4, 6, 5, 9, 7, 1, 10, 8\}$$

are

$$\begin{aligned} m_{max} = 5: \quad & \mathcal{S}_1 \equiv \{2, 4, 5, 9, 10\} \\ & \mathcal{S}_2 \equiv \{2, 4, 5, 7, 10\} \\ & \mathcal{S}_3 \equiv \{2, 4, 5, 7, 8\} \end{aligned}$$

These are not exhaustive!

Minimal Longest Ascending Subsequence (MLAS)

- MLAS is the longest ascending subsequence with the *smallest terminal element*

$$\mathcal{X}_{\mathcal{B}} = \{3, 2, 4, 6, 5, 9, 7, 1, 10, 8\}$$

$$m_{max} = 5: \quad \begin{aligned} \mathcal{S}_1 &\equiv \{2, 4, 5, 9, 10\} \\ \mathcal{S}_2 &\equiv \{2, 4, 5, 7, 10\} \\ \mathcal{S}_3 &\equiv \{2, 4, 5, 7, 8\} \end{aligned}$$

- $\mathcal{S}_3 < \mathcal{S}_2 < \mathcal{S}_1$ because $8 < 10$ and $7 < 9$

- $\mathcal{S}_3 \equiv \{2, 4, 5, 7, 8\}$ is the (unique) MLAS of $\mathcal{X}_{\mathcal{B}}$

Packet sequence 'orderedness' metric Ω

- $\mathcal{X}_{\mathcal{B}} = \{3, 2, 4, 6, 5, 9, 7, 1, 10, 8\}, \quad N = 10$
- $m_{max} = 5: \quad \mathcal{S}_3 \equiv \{2, 4, 5, 7, 8\}$ is MLAS
- Define 'orderedness' metric as

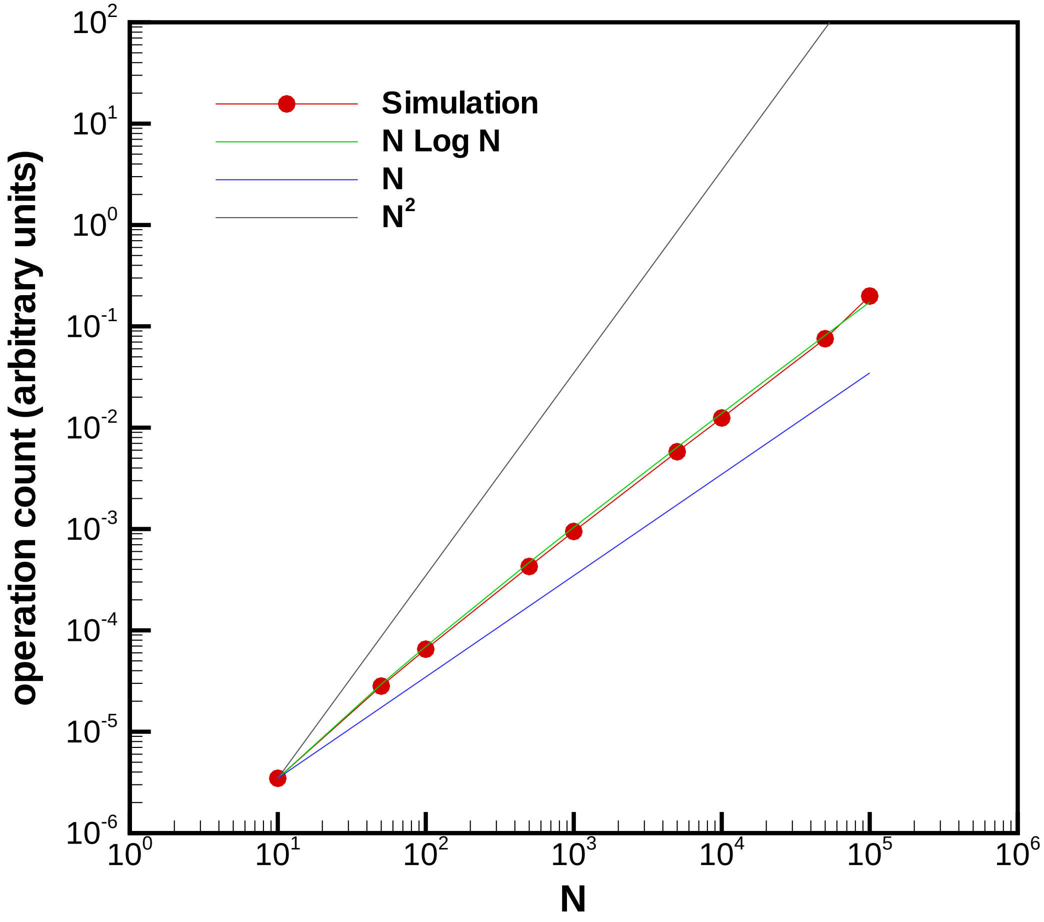
$$\Omega \equiv \frac{m_{max}}{N}$$

$$\text{our example}: \quad \Omega = \frac{5}{10} = 0.5$$

- Number of 'packet moves' to sort sequence $= N - m_{max}$

MLAS algorithm

- See: http://www.tiac.net/users/cri/mlas.html
- Sequence of N elements $\mathcal{X}_N = \{p_1, p_2, ...p_N\}$
- Suppose we have MLAS for $\mathcal{X}_{K-1} = \{p_1, p_2, ...p_{K-1}\}$
  $K - 1 < N$
- Use $[\mathcal{X}_{K-1}; p_K] \rightarrow$ MLAS of $\mathcal{X}_K$ (with binary search)
- Repeated application $\rightarrow$ MLAS of $\mathcal{X}_N \rightarrow \Omega = \frac{m_{max}}{N}$
- Order $(N \log N)$ operations and Order(N) storage



MLAS algorithm cost versus length of sequence N.

Summary

- Proposed packet orderedness parameter based on MLAS

$$\Omega = \frac{m_{max}}{N}$$

m_{max} = length of MLAS

- $\Omega = 1 \rightarrow$ packets received in order sent

- Ω determined in Order($N \log N$) operations with Order(N) memory

- Implementation; the algorithm has been implemented on CQOS equipment