Multi-source Network Error Correction

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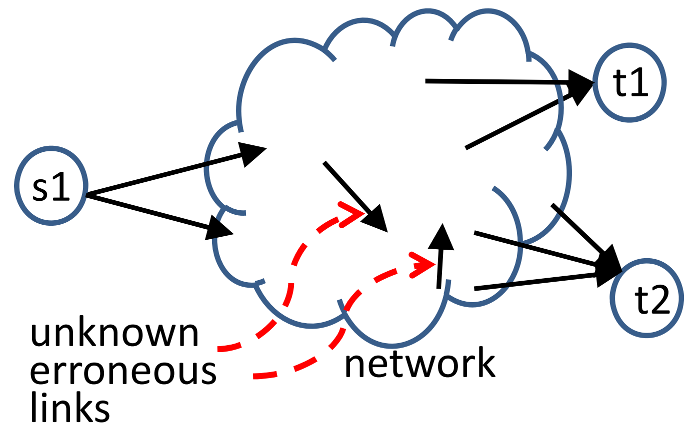
Code On

Background: Network error correction coding

- Network coding for reliable communication under arbitrary errors on an unknown subset of links/packets
- Generalization of both error-free network coding and classical error correction coding



Classical error correction coding

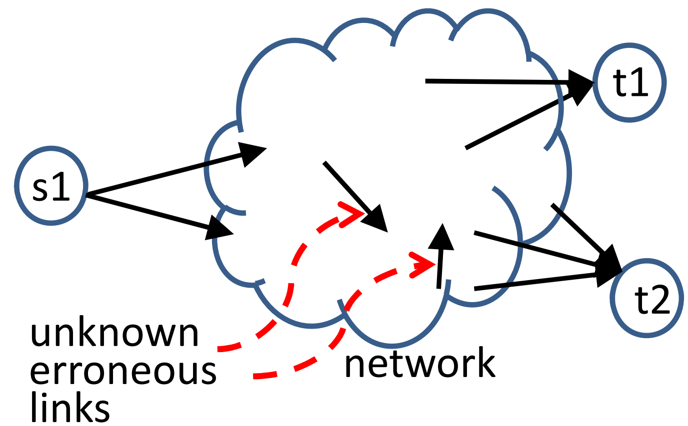
- Code across bits/symbols with redundancy, to combat bit/symbol errors
- Point-to-point, single-source single-sink

Network error correction coding

- Code across network links/packets with redundancy to combat link/packet errors
- More complicated topologies, may have multiple terminals

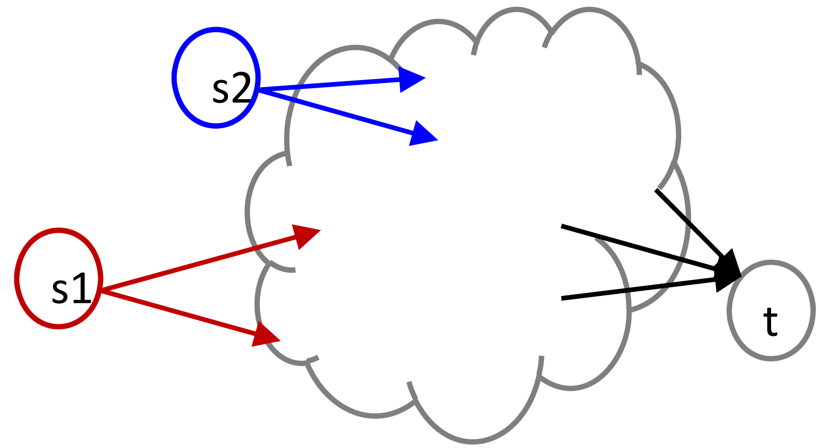
Background: Network error correction coding

- Problem originally studied for the simplest case [Cai & Yeung 03, 06]:
  - Single source multicast, equal capacity links/packets, any z may be erroneous
  - Error correction capacity = *mincut* - 2z
  - Various low-complexity capacity-achieving network codes combining end-to-end error correction with random linear network coding (RLNC) in the network, e.g. [Cai & Yeung 06, Jaggi et al. 08, Koetter & Kschischang 08]



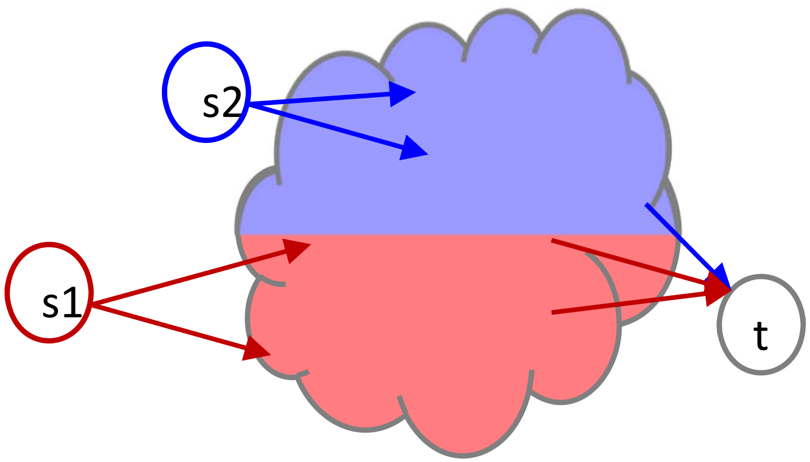
Multiple-source multicast

- Sources with independent information



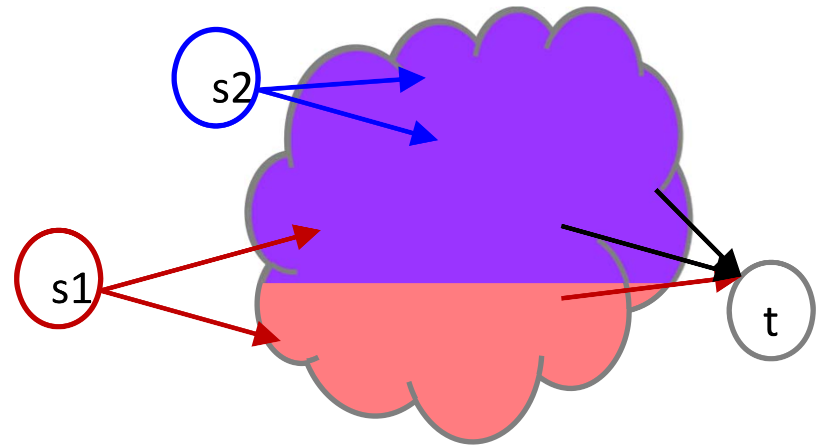
Multiple-source multicast

- Sources with independent information
- We could partition network capacity among different sources…



Multiple-source multicast

- Sources with independent information
- We could partition network capacity among different sources…
- But could rate be improved by coding across different sources? To what extent can different sources share network capacity?
- Challenge: owing to the need for coding across sources in the network and independent encoding at sources, straightforward extensions of single-source codes are suboptimal
- Previous work: code construction in (Jafari, Fragouli & Diggavi 08) achieves capacity for $C1+C2=C$



- Coherent case:
  - network topology is known
- Noncoherent case:
  - network topology is unknown
  - captured in RLNC header, which can also suffer errors

Capacity region

- ***Theorem**: The coherent and non-coherent capacity region for multi-source multicast under any z link errors is given by the cut set bounds*

$$\sum_{i \in S} r_i \leq m_S - 2z, \forall S \subseteq U$$

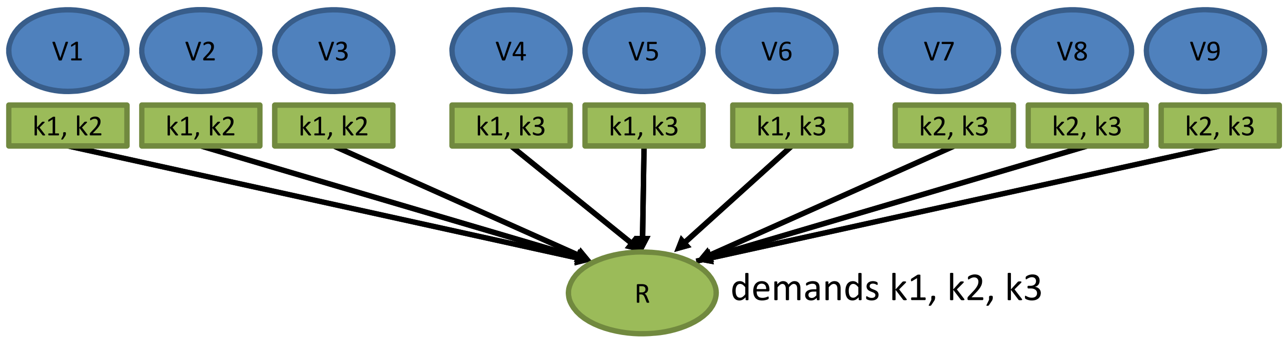
  - U = set of source nodes
  - m_S = min cut capacity between sources in subset S of U and each sink
  - r_i = rate from the i^{th} source

- Implication: redundant capacity can be fully shared across sources
- This is achieved by novel distributed error correction codes employing RLNC in the network
  - Construction involving rank metric codes in nested finite fields
- Note that RLNC in error-free scenario enables capacity sharing across multicast receivers; in the error-correction scenario RLNC additionally enables capacity sharing across sources

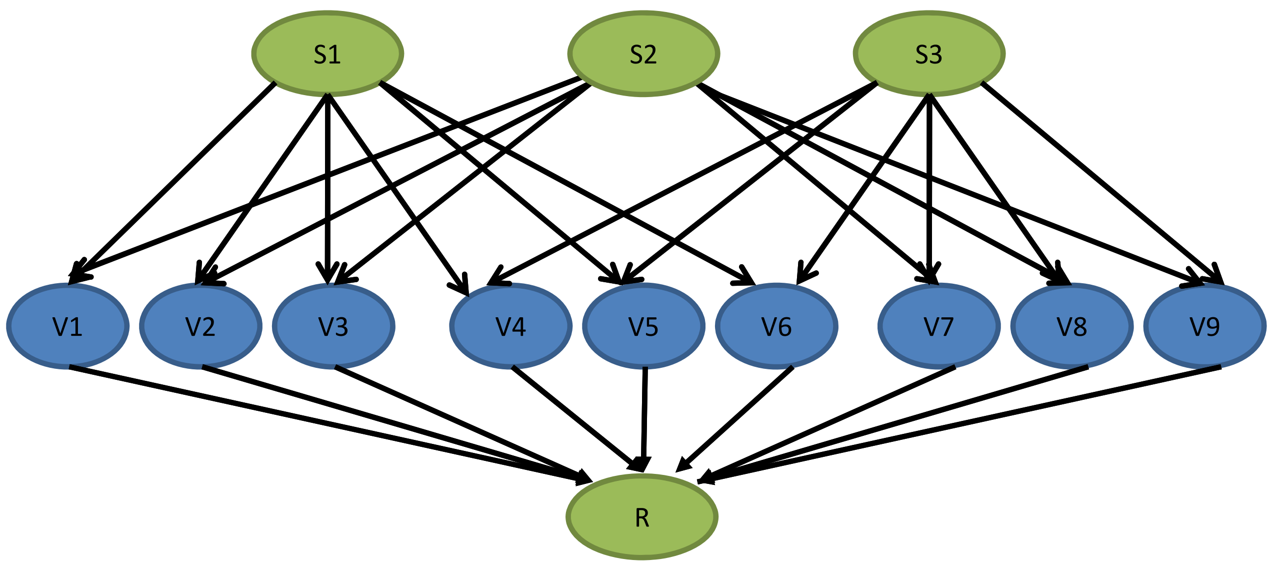
Dikaliotis, Ho, Jaggi, Vyetrenko, Yao, Effros, Kliewer and Erez, "Multiple access network information flow and correction codes," IT Transactions 2011.

Robust distributed download

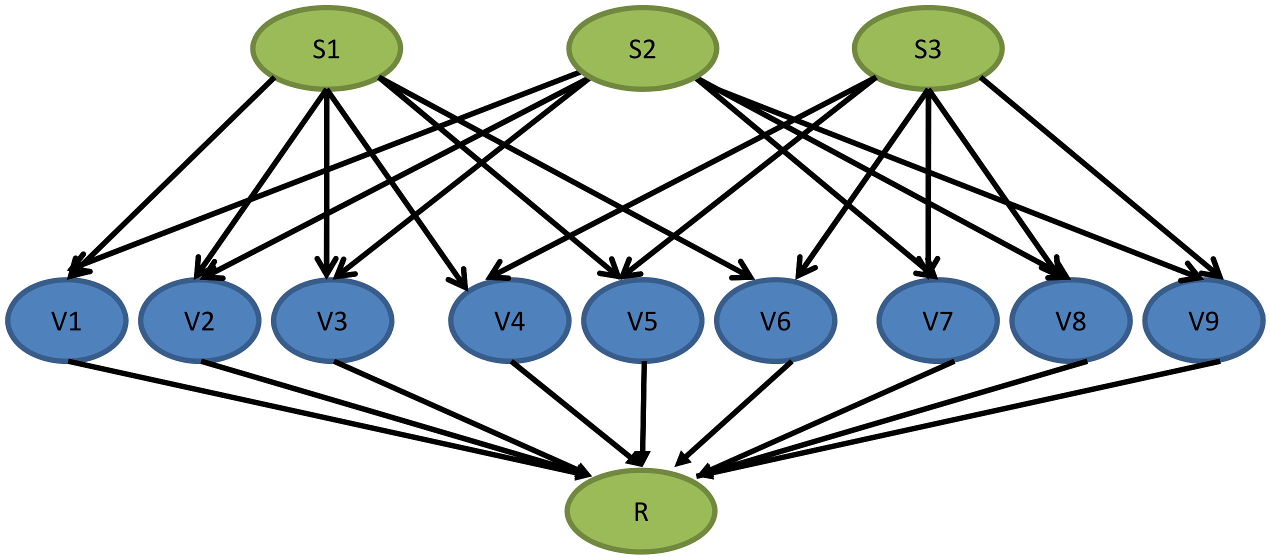
- Problem: Reliable download from distributed storage nodes/peers who each have a subset of the demanded information



- Protecting against a subset of erroneous nodes/transmissions is equivalent to network error correction on a simple multiple access network



Multi-source Reed Solomon code



- We can construct a multi-source network code in which vectors of received symbols correspond to codewords from a classical single-source Reed Solomon code
- We have proven that our construction achieves any rate vector in the capacity region for up to three sources, and numerical investigation suggests more general optimality
- Allows application of off-the-shelf Reed Solomon decoders

Halbawi, Ho, Yao and Duursma, "Distributed Reed Solomon codes for simple multiple access networks," ArXiv 2013.

Thank You!