The Algebraic Eraser: a linear asymmetric protocol for low-resource environments

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Algebraic Eraser

- Asymmetric key agreement protocol
- Designed for low-cost platforms with constrained computational resources
  - RFID
  - Bluetooth
  - NFC
  - “Internet of Things”
- Complexity scales *linearly* with desired security level, unlike RSA, ECC.
## AE Performance vs ECC

### $2^{128}$ Security level (AES–128)

<table>
<thead>
<tr>
<th>ECC 283</th>
<th>AE $B_{16}, F_{256}$</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>164,823</td>
<td>29,458</td>
<td>4,855,355,934</td>
</tr>
<tr>
<td>85,367</td>
<td>77,858</td>
<td>6,646,503,866</td>
</tr>
<tr>
<td>70,469</td>
<td>195,382</td>
<td>13,768,374,158</td>
</tr>
</tbody>
</table>

Wtd. Perf. is Weighted Performance (clock cycles $\times$ gate count) and represents time and power usage. Gate counts are for 65nm CMOS. ECC data taken from *A Flexible Soft IP Core for Standard Implementations of Elliptic Curve Cryptography in Hardware*, B. Ferreira and N. Calazans, 2013 IEEE 20th International Conference on Electronics, Circuits, and Systems (ICECS), 12/2013.
Overview of AE

- The AE key exchange is a nonabelian Diffie–Hellman exchange.
- The underlying algebraic structure is not \((\mathbb{Z}/N\mathbb{Z})^\times\) or \(E(\mathbb{F}_q)\), but rather
  - \(M_n(\mathbb{F}_q)\) (\(n \times n\) matrices over \(\mathbb{F}_q\)),
  - \(B_n\) (the braid group on \(n\) strands).
- Private keys: a pair \(R = (m, \mu)\) of a matrix and braid.
- Public keys: a pair \(P = (M, \sigma)\) of a matrix and a permutation in \(S_n\).
- Each user also knows a fixed ordered list of elements of \(\mathbb{F}_q\) (\(T\)-values).
- The shared secret: same kind of pair as the public key.
Overview of AE

- The security level depends on $n, q$ and the lengths of the private braids (and scales linearly with the lengths of the braids).

- The (maximum) security level for AE is $n \cdot \lg q$, not $(\lg q)/2$ as in ECC. In particular one can use moderately sized finite fields, not multiprecision finite fields.

- The hard computational problem underlying AE takes place in the braid group $B_n$, and is known as the *Simultaneous conjugacy separation search problem*. This is not the same computational problem underlying earlier braid group schemes, and AE is not “Braid Group Cryptography.”
Braids

A braid on \( n \) strands is a collection of \( n \) entangled strings.

We can represent a braid by a \textit{left-right crossing sequence} of signed nonzero integers \( i_1 i_2 \cdots i_k \), ("Artin generators") each of which lies between \(-n\) and \( n\).

- A positive integer \( i \) means "cross the \( i \)th strand \textit{under} the \((i + 1)\)st strand."
- A negative integer \(-i\) means "cross the \( i \)th strand \textit{over} the \((i + 1)\)st strand."

\[
1\ 2\ 3\ 1\ 2\ 1\ 3\ \ -3\ \ -2\ \ -2\ 1\ \ -3\ \ -1
\]
**E-multiplication**

E-multiplication is an action of $B_n$ on $M_n(\mathbb{F}_q)$

- Each Artin generator determines an $n \times n$ sparse matrix, a *colored Burau matrix*.
- This matrix depends on the $T$-values (the fixed set of elements in $\mathbb{F}_q$), but the correspondence between generators and matrices changes as one moves down the braid in the private key.
- This nontrivial permuting of the $T$-values is the “eraser” part of the construction. Effectively it masks the map between braids and matrices.
- E-multiplication is how the public keys are produced from the private data: $P_A = m_A \ast \mu_A$, $P_B = m_B \ast \mu_B$ ($A = Alice$, $B = Bob$).
Shared secret computation

- Bob and Alice take each others public keys $P_A = (M_A, \sigma_A), P_B = (M_B, \sigma_B)$, and multiply their private matrices $m_A, m_B$ against them.

- Then they $E$-multiply the result by their braids $\mu_A, \mu_B$:

  $$S_A = P_B m_A \ast \mu_A, \quad S_B = P_A m_B \ast \mu_B.$$ 

- We have $S_A = S_B$.

Many details have of course been elided, for example how one chooses the matrices and braids.
Thank You!

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