Updates on NADA: Stability Analysis and Impact of Feedback Intervals

draft-ietf-rmcat-nada-02

Xiaoqing Zhu, Rong Pan, Michael A. Ramalho, Sergio Mena de la Cruz, Paul Jones, Jiantao Fu, Stefano D’Aronco, and Charles Ganzhorn

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Outline

• Update on draft -02
• Stability analysis of NADA feedback control loop
• Numerical results on NADA with varying feedback intervals
• Simulation results on NADA with varying feedback intervals
• Summary and next steps
Changes in Draft -02

- No algorithm changes
- Added a section on feedback requirements of NADA in Sec. 5.3
- Addressed review comments from Stefan and Zahed (Thanks!)
- Minor adjustment in notations, fixed various errors and typos.
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• Update on draft -02

• **Stability analysis of NADA feedback control loop**
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Simplifying Assumptions for Stability Analysis

- Considers only gradual rate update mode, w/o packet losses or marking: \( x_{\text{curr}} = d_{\text{queue}} \)

- Ignores effect of 15-tap minimum filtering

- Rate update equation reduces to (see Eq(5)-(7) in draft):

\[
\begin{align*}
    r_i &= r_{i-1} - \kappa \frac{\Delta}{\tau} \frac{x_i - x_o}{r_{i-1}} r_{i-1} - \kappa \eta \frac{x_i - x_{i-1}}{r_{i-1}} \\
    x_o &= PRIO \frac{R_{MAX}}{r_o} x_{\text{ref}}
\end{align*}
\]
Feedback Control Loop in Laplace Transform

System at equilibrium:

\[ r_o = PRIO \frac{x_{ref}}{x_o} R_{max} \]

For single flow:

\[ r_o = C \]
Open Loop Transfer Function

\[ G(s) = -\frac{r_o}{C} \frac{1 + \eta s \tau}{1 + \frac{\tau}{\kappa x_o} s \tau} \frac{e^{-sRTT}}{sx_o} \]

At low frequency, \( s \to 0 \)
\[ G(s) \approx -\frac{r_o}{C} \frac{RTT}{x_o} \]

Bandwidth sharing proportional to \( PRIOR_{max} \)

At high frequency, \( s \to j\infty \)
\[ G(s) \approx -\kappa \eta \frac{r_o}{C} \frac{RTT}{\tau} \frac{e^{-sRTT}}{sRTT} \]

Guarantees stability for
\[ \kappa \eta \frac{RTT}{\tau} < \frac{\pi}{2} \quad \text{and} \quad \eta \tau \gg 1 \]
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• Open Issues and next steps
Bode Diagram with Gain/Phase Margins

Bode Diagram

$G_m = 15.1$ dB (at $11.5 \text{ rad/s}$), $P_m = 50.7$ deg (at $2.2 \text{ rad/s}$)

Propagation delay = 50ms
Bottleneck BW = 1Mbps

Feedback interval @ 100ms
Feedback interval @ 1s
Bode Diagram with Gain/Phase Margins

Bode Diagram
\[ G_m = 13.8 \text{ dB} \text{ (at } 9.87 \text{ rad/s}) \], \[ \phi_m = 49.3 \text{ deg} \text{ (at } 2.2 \text{ rad/s}) \]

propagation delay = 50ms
bottleneck BW = 1Mbps

feedback interval @ 100ms
feedback interval @ 1s
Step Response of Closed-Loop System

propagation delay = 50ms
bottleneck BW = 1Mbps
Step Response with Feedback Interval @ 100ms

- Propagation delay = 50ms
- Bottleneck BW = 1Mbps
Step Response with Feedback Interval @ 200ms

- Propagation delay = 50ms
- Bottleneck BW = 1Mbps
Step Response with Feedback Interval @ 500ms

Propagation delay = 50ms
Bottleneck BW = 1Mbps
Settling Time vs. Feedback Interval

propagation delay = 50ms
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Propagation Delay @ 50ms, Feedback Interval = 20ms
Propagation Delay @ 50ms, Feedback Interval = 50ms

NS2: physical link rate change

NS3: time-varying background UDP flow
Propagation Delay @ 50ms, Feedback Interval = 100ms

**NS2: physical link rate change**

**NS3: time-varying background UDP flow**
Propagation Delay @ 50ms, Feedback Interval = 200ms
Propagation Delay @ 50ms, Feedback Interval = 500ms

NS2: physical link rate change

NS3: time-varying background UDP flow
Propagation Delay @ 50ms, Feedback Interval = 1s

NS2: physical link rate change

NS3: time-varying background UDP flow
Propagation Delay @ 50ms, Feedback Interval = 2s

NS2: physical link rate change

NS3: time-varying background UDP flow
Propagation Delay @ 150ms, Feedback Interval = 20ms

NS2: physical link rate change

NS3: time-varying background UDP flow
Propagation Delay @ 150ms, Feedback Interval = 200ms

NS2: physical link rate change

NS3: time-varying background UDP flow
Propagation Delay @ 150ms, Feedback Interval = 2s

**NS2: physical link rate change**

**NS3: time-varying background UDP flow**
Convergence Time vs. Feedback Interval

NS2: Transition after $t=120s$

Overhead ~ 1.6% @ 1Mbps

NS3: Transition after $t=120s$
Summary and Next Steps

- Guaranteed stability of NADA feedback control loop for RTT < 500ms
- Qualitatively matching results from numerical analysis and simulation results:
  - Remains stable for sub-second feedback intervals
  - System response slows down with increasing feedback intervals
  - Recommended feedback interval at 100ms — tradeoff between overhead and response speed
- Next steps:
  - Investigate different convergence behavior with different BW changing mechanisms;
  - Study system stability with varying parameter choice and network settings
Backup Slides
Derivation of Laplace Transfer Function for Gradual Rate Update

Consider small perturbation around equilibrium: \( \delta x = x_i - x_o, \quad \delta r = r_i - r_o \)

\[
\frac{\delta \dot{r}}{\tau} = -\frac{\kappa}{\tau} \left[ \frac{\delta x r_o}{\tau} + \frac{x_o \delta r}{\tau} + \eta \delta \dot{x} r_o \right]
\]

In Laplace domain:

\[
\frac{\kappa x_o}{\tau^2} (R(s) + \frac{\tau^2}{\kappa x_o} s R(s)) = -\frac{\kappa r_o}{\tau^2} (X(s) + \eta \tau s X(s))
\]

\[
\frac{R(s)}{X(s)} = -\frac{r_o}{x_o} \frac{1 + \eta \tau}{1 + \frac{\tau}{\kappa x_o} s \tau}
\]