Memory-Hard Functions

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Theory: Quo Vadis?

Goal 1: Inform future research direction aiming it in a “useful” direction.

Goal 2: Raise awareness of potential implications of recent results for Password-hashing standardization.

Some example questions to keep in mind…

1. Computational Model: Too weak / strong for security statements / attacks? If so what is wrong?
2. Complexity Measures: Too weak / strong for security statements / attacks?
3. Statements: Are the type of statements being proven relevant to practice? What more would we like to know?

MHF a la Percival

- Observation: Computation is cheaper for custom hardware (e.g. ASICs) then general purpose CPUs.
- Goal: Functions which require as much memory as possible for a given number of computational steps even in parallel.

  $\Longrightarrow$ Decrease "evaluations/second per dollar" advantage of ASICs.
- Recall: Area x Time (AT) complexity of a circuit evaluating $f \approx$ dollar cost per unit of rate (rate = # of f evaluations / sec).
- Percival: Since high speed memory is expensive in circuits replace "area" with "space" (i.e. memory).

MHF a la Percival

Definition [Per09]:

An MHF is a function f with hardness parameter n such that f_n:

1. can be computed on a Random Access Machine (RAM) in T*(n) time.
2. can not be computed on a Parallel RAM (PRAM) with S(n) space and processors and T (n) time such that $T(n) \times S(n) = O(n^{2-c})$ for some $c > 0$.

Data-(in)dependence

- Is the honest evaluation algorithms memory access pattern input-dependent?
  - Yes: data-dependent MHF (dMHF). Example: scrypt, Argon2d.
  - No: data-independent MHF (iMHF). Example: Argon2i, Balloon Hashing.

iMHF Advantage: Implementations easier to secure against certain timing attacks.

Overview

1. Intuitive goals of an MHF.
2. Theory for proving security.
3. Attacking an MHF.

Computational Model

Problem: Proving complexity lower-bounds is hard.

Fortunately almost all proposed MHFs based on compression functions.

Idea: Use (Parallel) Random Oracle Model.

Parallel Random Oracle Model

- Computational Model: PROM
  - Algorithms **A** invoked iteratively.
  - At iteration *i* do:
    1. Get input state s_{i-1} (state = arbitrary bit-string).
    2. Perform arbitrary computation.
    3. Make one <u>batch</u> of queries to $\mathcal{H}$. (i.e. make parallel queries.)
    4. Perform arbitrary computation.
    5. Output new state s_i.
  - Set s_0 to be the input to the computation.
  - Repeat until **A** produces a special output state s_z = result of computation.

Parallel Random Oracle Model

Intuition: Good for proving security because…

1. Rather permissive $\Rightarrow$ security proofs carry more weight.
   - Arbitrary non-RO dependent computation allowed for free at each step.
   - Memory only measured between calls to RO.
   - Any PRAM algorithm is a PROM algorithm (at no added cost).

2. Proving <u>exact</u> lower-bounds with <u>reasonable constants</u> is tractable.

ST-Complexity

- Computational Model: PROM
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  - Repeat until **A** produces a special output state s_z = result of computation.

- Cost(execution) := $\max_{i \in [z]} |s_i| \times z$

computation time

bit length largest state

Sanity check? "Cost(execution) is high ⇒ AT(execution) is high
⇒ expensive to implement in ASIC or FPGA."

ST-Complexity of a Function

- Complexity of an algorithm **A** on input x:

$$ST(\mathbf{A},x) \geq c \Leftrightarrow \Pr[\, ST(exec(\mathbf{A}^H(x)) \geq c] \geq 1\text{- negligible}$$

over the choice of RO.

Intuition: "On input x algorithm A almost always runs with ST-complexity at least c."

- Complexity of a function f^H:

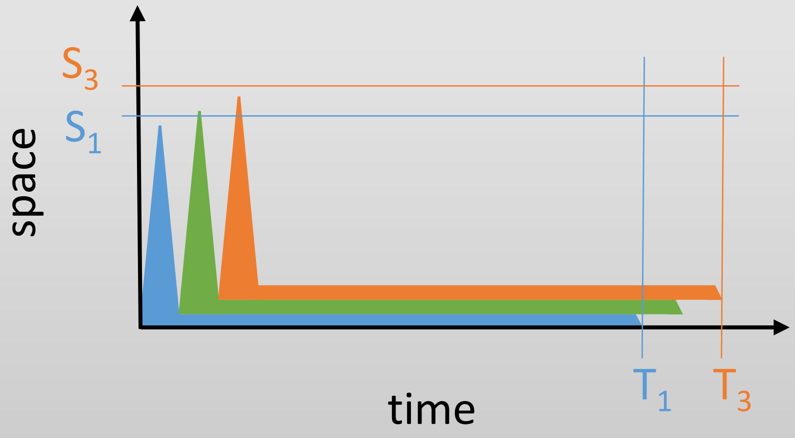
$$ST(f) = \min_{\mathbf{A},x} \{\, ST(\mathbf{A},x) \,\}$$

minimum over all alg. **A** and inputs x computing f(x).

Intuition: ST complexity of the best algorithm computing *f* on its favorite input x.

Amortized and Parallelism

- Problem: for parallel computation ST-complexity can scale badly in the number of evaluations of a function.



$$ST_1 = S_1 \times T_1 \approx S_3 \times T_3 = ST_3$$

cost of computing *f* once

cost of computing *f* three times

In fact ∃ function *f* (consisting of n RO calls) such that: $ST\left(f^{\times\sqrt{n}}\right) = O(ST(f))$

Amortized ST-Complexity of a Function

- Amortized ST-complexity of a function f

Intuition: "The ST-complexity per I/O pair of the best evaluation algorithm for f running on its favorite set of inputs."

$$\mathrm{aST}(f) = \min_{m\in\mathbb{N}} \frac{ST(f^{\times m})}{m}$$

- Sanity check? "If aST(f) is large $\Longrightarrow$ Implementing brute-force attack in an ASIC is expensive."

Examples of Results

- Argon2i (and Balloon Hashing) security proofs:
  - For any choice of mem-cost σ and time-cost τ = 1

$$aST(\text{Argon2i}_{\sigma,\tau}) \geq \Omega(\sigma^{1.666})$$

with probability at least $1-o(\sigma^{-3})$ over choice of RO and salt.

Note: larger τ can only give worse complexity because

Recall: In practice $\sigma \approx 2^{24}$ for 1GB of memory $\Longrightarrow \sigma^{-3} \approx 2^{-68}$

- Construct an iMHF f_n with:
  1. f_n computable in n Time and n Space in (sequential) ROM.
  2. $aST(f_n) = \Omega\left(\frac{n^2}{\log n}\right)$ in the PROM for all "reasonable" adversaries.

"completeness"

"security proof"

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When is an Evaluation Algorithm an “Attack”?

Intuitive Answer: An evaluation algorithm **A** is an “attack” if it has lower complexity then the honest algorithm **N**.

More fine grained: Quality(**A**) = complexity(**A**) / complexity(**N**).

But which “complexity”?

- aST considers only memory. What about cost of implementing RO?
- aST ≈ cost of building ASIC. What about cost of <u>running</u> device?

Two Stricter Complexity Measures

1) Amortized-Area/Time Complexity (a-AT) ≈ cost of building ASIC.

- Area: accounts for memory needed on chip <u>and RO cores</u>.

2) Amortized-Energy (aE) Complexity ≈ cost of running ASIC.

- Accounts for electricity consumed while storing values and RO evaluations.

amortized-AT Complexity

- Recall PROM: At iteration i make batch of queries q_i and store state s_i.
- Initial Idea: $\quad aAT(execution) := \max_i(|s_i|) + \max_j(q_j)$.

# of memory cells needed to run execution.

# of RO cores needed to run execution.

amortized-AT Complexity

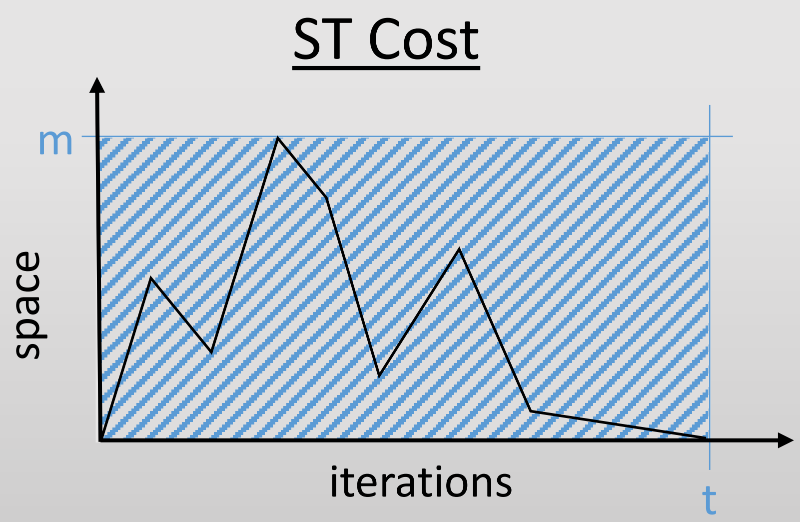
- Recall PROM: At iteration i make batch of queries q_i and store state s_i.
- Initial Idea: aAT(execution) := $\max_i(|s_i|) + \max_j(q_j)$.
- Problem: Storing 1-bit requires much less area then implementing, say, SHA1.
- Solution:

  "Core-memory area ratio" R := area(1-bit-storage) / area(RO)
- Parametrized Complexity:

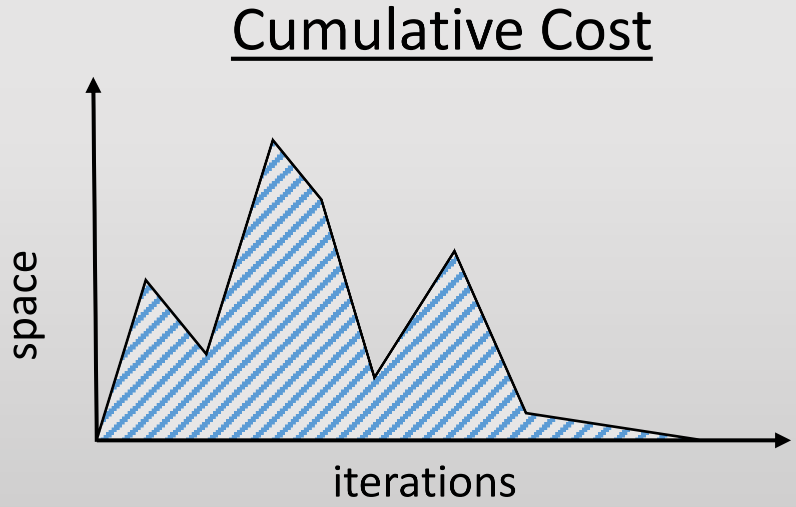
$$aAT_R(\text{execution}) := \max_i(|s_i|) + R*\max_j(q_j)$$

Energy Complexity

- Intuition: Only pay for memory that is being actively used.
- Idea: Define the complexity to be area under the “memory curve”.



$\longmapsto$



Energy Complexity

- Similarly for RO calls: Only pay for actually making a call.
- Unit of time: “tock” = time it takes to evaluate the RO.
- Unit of measure: milli-Watt-tock (mWt) = Electricity required to store 1-bit for one tock.
- “Core-memory energy ratio” R’ = mWt requires to evaluate the RO on one input.

$$aE_{R'}(\text{execution}) := \sum |s_i| + R' \times |qi|$$

Asymptotic Example: Argon2i

- [AB16] For mem-cost σ and time-cost τ such that $\sigma \times \tau = n$

$$\mathrm{aAT_R(Argon2i)} = O(n^{1.75} \log n + Rn^{1.25})$$

$$\mathrm{aAT_R(Honest\text{-}Alg)} = \Omega\left(\frac{n^2}{\tau} + Rn\right)$$

on expectation over the choice of salt and RO.

- Same for energy complexity.
- Similar (or stronger) asymptotic attacks for Catena-BRG, Catena-DBG, Balloon Hashing 1, 2 & 3, Lyra2, Gambit, Rigv2.

Asymptotic Example: General Upper-Bound

- Any MHF making n calls to a RO has complexity

$$\mathrm{aAT}_R(\mathrm{Argon2i}) = O\left(\frac{n^2}{\log n} + R \times n\right)$$

$\Longrightarrow$ At least in principle Percival's goal of n^2 is impossible for an iMHF.

Exact Example: Argon2i

- For mem-cost σ and time-cost τ such that σ × τ = n

$$\mathrm{aAT}_{\mathrm{R}}(\mathrm{Argon2i}) \leq 2n^{1.75}\left(5 + \frac{\log n}{2} + \tau + \frac{R}{n^{.75}} + \frac{R}{n^{.5}} + \frac{2R}{n}\right)$$

- Similar for $\mathrm{aE}_{\mathrm{R'}}$(Argon2i)

Exact Example: Argon2i

- What does this mean for standardizing Argon2i?

- Some arguments for “This is only a theoretical attack.”
    1. aAT complexity doesn’t charge for computation not involving a call to the RO so real complexity may be far bigger.
    2. Setting $n=2^{24}$, $R=3000$ and $\tau \geq 2$ gives worse complexity than honest alg.
    3. It needs unrealistic amounts of parallelism.

- First: besides calling RO practically no further computation done (In fact: potentially less than honest algorithm…)

Exact Example: Argon2i

- Second: Set $n=2^{24}$, $R=3000$ and $\tau \geq 2$ then this is <u>not</u> an attack.
- Conceptually: By increasing τ we increase computation while keeping memory the same. Intuitively it becomes “less memory-hard”.
- No attempt … n made … e:
  - for specific parameter ranges
  - minimizing exact security (vs. asymptotic)

1GB Mem

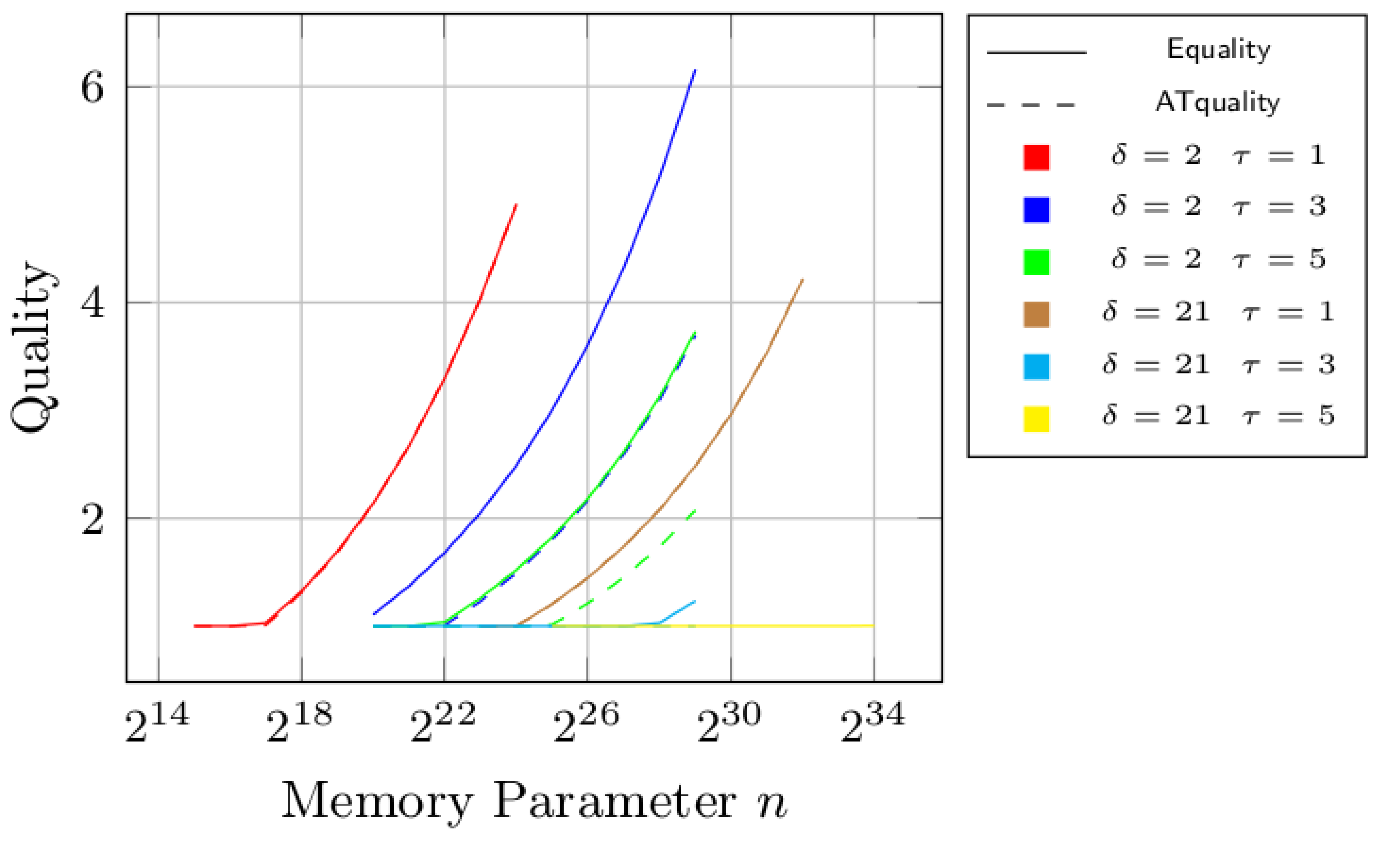
Passes over memory

Optimizing Analysis for Concrete Parameters

Argon2: indegree δ = 2

- For 1GB memory ($n=2^{24}$) actually need τ ≥ 6.
- For each quadrupling of memory need 1 more pass on memory.

Further optimizations of the <u>analysis</u> possible? Most likely…



(a) Argon2i and SB

Third: Can Actually Build This Attack?

- Example: Compute 2^{12} instances in time 2^{25}.
- Recall: In Argon2i RO = Blake-512 $\approx .1\ \mathrm{mm}^2$.
- Layout: 1 “big” ASIC + 256 “light” ASICs.
- Big ASIC: 2^{12} Blake-512 Cores $\approx 410\ \mathrm{mm}^2$.
- Total memory on device $\approx$ 50 GB.
- These aren’t unrealistic requirements for an attacker with decent budget...

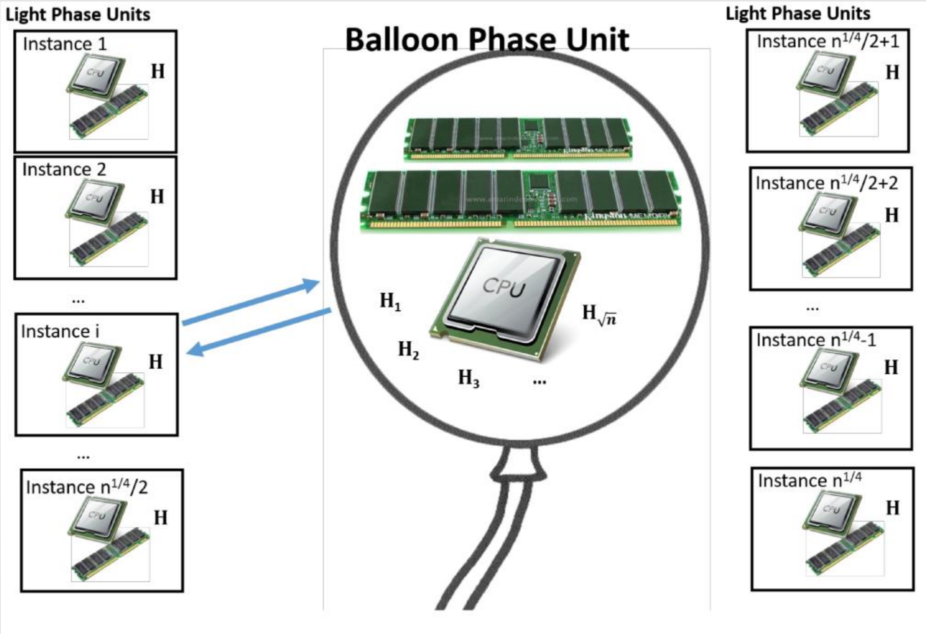


Figure 2: Argon2i: Attack Architecture

Conclusions

Argon2i

- In it’s current form attack is neither “apocalyptic” nor “only theoretical”.
- Could it improve: my opinion is “very likely yes” both asymptotically and exact.
    - See history of block ciphers and hash functions. Attacks tend to improve…
- What else could we even use?
    - Balloon Hashing?
    - Something new?

Theory: Quo Vadis?

- You tell me!
    - What do you think of the PROM?
    - How about aAT and Energy complexity?
    - Are the statements being proven somewhat meaningful?
    - What else could theory try to consider?

Questions? Comments?